UNIVERSITÉ DE MONTRÉAL

CENTRE DE RECHERCHE SUR LES TRANSPORTS

CASE POSTALE 6128, SUCCURSALE A, MONTRÉAL, QUÉBEC, H3C 3J7

CRT-597

IS TRIP DISAGGREGATION WORTHWHILE IN MODELS OF AIR TRAVEL DEMAND?

by

Richard Laferrière*

* Assistant Professor at the Concordia University. This research was done while I was at the Centre de recherche sur les transports, Université de Montréal.

I am firstly indebted to Marc Gaudry for his helpful comments during the duration of this project and, secondly to Marcel Dagenais and Sophie Mahseredjian for their suggestions at several stages of this research. I also thank James McIntosh and Guy Picard for their helpful comments. The collaboration of the air carriers Air Canada and CP Air was essential for the elaboration of this project. Some people from these carriers to which I am grateful are: Samuel Arnold, Brent Statton, Mrs. Donaldsen, Arthur Fischer, Kent James, Dave Johnston. This research was supported in part by Transport Canada, the F.C.A.R. program of the Quebec government, the Social Sciences and Humanities Research Council of Canada, and the Natural Sciences and Engineering Research Council of Canada.

Université de Montréal

Centre de recherche sur les transports - Publication #597

July 1988

Dépôt légal - 3e trimestre 1988

ABSTRACT

This paper develops the ANITA model (Air Network Itinerary and Trip Aggregation) to explain the air travel demand between several Canadian city pairs. The parameters are estimated with data on the individual movements of travelers with Air Canada and CP Air during 1983. The information about individual trip movements allows four innovative dimensions of the demand model:

i) the formulation of the itinerary concept, defined for each origin-destination pair, as a set of paths actually used that have the same price and travel time characteristics;
ii) in some cases the identification of the travelers' place of residence;
iii) the treatment of two kinds of trips: the round trips and the one-way trips;
iv) the explicit formulation of travelers' choice among the itineraries of each origin-destination pair and the utilization of all of the observed characteristics of the itinerary set (or the service level) to explain the level of the aggregated demand for a market.

These four dimensions, all useful to explain the interurban air travel demand, allow (render possible) the demonstration of the benefits of distinguishing between round trips and one-way trips instead of explaining the origin-destination trips.

Key words: air travel demand, origin-destination trip, round trip, one-way trip, path, itinerary, aggregation.

RESUME

Cet article développe le modèle ANITA (Agrégation Nouvelle des Itinéraires de Transport Aérien) de la demande des déplacements aériens des personnes entre différentes paires de villes canadiennes. Les paramètres sont estimés à partir de données sur les déplacements irdividuels par Air Canada et CP Air en 1983. Le modèle a quatre dimensions innovatrices. L'information recueillie pour les déplacements individuels permet:

i) d'introduire le concept d'itinéraire défini comme un ensemble de chemins utilisés à l'intérieur d'une paire origine-destination possédant les mêmes caractéristiques de coût et de temps;
ii) dans certains cas, d'identifier le lieu de résidence des individus;
iii) un traitement de deux types de voyages: les allers-simples et les allers-retours;
iv) la formulation explicite du choix des voyageurs entre les itinéraires d'un marché et la prise en compte des caractéristiques de l'ensemble des itinéraires (ou niveau de service) pour expliquer le niveau de la demande d'un marché.

Ces quatre dimensions, toutes utiles à l'explication de la demande interurbaine par avion, permettent de démontrer qu'il est utile de connaître l'origine réelle des déplacements plutôt que d'expliquer simplement les deplacements origine-destination.

Mots-clés: demande aérienne de transport, déplacement origine-destination, aller-retour, aller-simple, chemin, itinéraire, agrégation.

CONTENTS

Introduction....4
1 The Air Network and Itinerary Trip Aggregation (ANITA) Model....5
1.1 FORMULATION AND DATA....5
1.2 SPECIAL CASES....7
1.3 ESTIMATION PROCEDURE....11
2 RESULTS....12
2.1 ELASTICITIES....12
2.2 INVERSE POWER TRANSFORMATION BINOMIAL LOGIT PARAMETERS....12
2.3 ANALYSES OF THE RESULTS FOR THE ANITA MODEL....15
2.4 GAINS FROM THE RECOGNITION OF THE TRAVELERS' PLACE OF RESIDENCE....16
CONCLUSION....17
REFERENCES....18
APPENDIX 1....20
APPENDIX 2....22

Table 1....13

INTRODUCTION

Many authors have used the air travel demand data in several fields: regulation (Trapani and Olson, 1981, 1982; De Vany, 1975), demand analysis (Lansig, 1961; Long, 1968; Ippolito, 1981; Brown and Watkins, 1968), demand forecasting (Sobienak, 1972), value of travel time (de Vany, 1974; Gronau, 1970), etc. However, it is common knowledge that these data do not include any indication of the travelers' place of residence.[1]

We propose to investigate the following question: Does the knowledge of the travelers' place of residence significantly influence the estimation of the air travel demand? In order to answer this question, we disaggregate the usual trip data into complete and incomplete travels. This allows generalization of all trip models found in literature and gives new meanings to network and socioeconomic variables. The untold improvements for air travel demand estimation from the trip data disaggregation are revealed by the comparaison of the general formulation to the nested ones.

The estimation of the aggregated demand for city pairs requires an aggregation procedure for various characteristics, such as travel times and prices, within a given market. The major difference between the advocated and the general method rests on the explicit recognition of the travelers' choice between several paths.

In Part one, we formulate the Air Network and Itinerary Trip Aggregation (ANITA) model and describe the data bank used to derive the complete and incomplete travels. In section 1.2, we discuss the general trip model and categorize the proposed trip models in literature. In section 1.3, we expose the econometric procedure to measure the advantages of specifying the travelers' place of residence. In Part 2, we discuss the results obtained from the estimation of the ANITA model and those from the traditional formulation. Finally, concluding remarks will then follow.

---

[1] "Because it is impossible to distinguish between travelers going to city A from city B and those returning to city A from city B,...". Long (1968, p. 200).

PART 1
The Air Network and Itinerary Trip Aggregation (ANITA) Model

1.1 FORMULATION AND DATA

Throughout the paper, we will use the following terms: travel, trip, round trip, one-way trip and path. Let us start by defining them. A travel may include one or more trips. If the destination of the last trip corresponds to the origin of the first trip, then the travel is completed[2]. A trip represents an individual decision to leave a city (origin) to go to another city (destination). Since an individual spends a significant amount of time at the origin and at the destination, the latter represents the traveler's choice, contrary to stopovers set by the carriers. A round trip is a complete travel with two trips. The trips included in the incomplete travels are the one-way trips. A trip from city i to city j requires the use of a particular path. A path is a specific choice among many possible legs. The attributes of each leg are: departure and arrival time, carrier, day of the week, fare, fare class, and departure and arrival cities. A trip itinerary corresponds to the set of paths having the same travel duration, waiting time, and fare.

The formulation of the ANITA model is expressed in equations (1) to (4). We assume the round trips by plane from city i ($T_{ij i}$) depends on: the level of service of markets ij (S_{ij}) and ji (S_{ji}), the car and bus attributes of a trip from i to j (M_{ij}) and from j to i (M_{ji}), and the socioeconomic variables of cities i and j (X_i, X_j, X_{ij}). The one-way trips from city i to city j (T_{ij}) are explained with similar variables. The share of trips between origin i and destination j with itinerary t (O_{ijt}) in trips between origin i and destination j (O_{ij}) depends on the characteristics of itinerary t (Z_{ijt}). Each of the random variable ϵ_{iji}, ϵ_{ij} and u_{ijt} is assumed identical and independently distributed. The level of service of market ij (S_{ij}) includes the characteristics of all the itineraries in this market (eq. 4). We purposely omitted the dimensions of the line vectors (X_i, M_{ij}, X_{ij}) and the column vectors (β, γ, θ) to facilitate the communication between the author and the reader:

$$T_{iji} = \beta_0 + (S_{ij} + S_{ji})\,\beta_1 + X_i\,\beta_2 + X_j\,\beta_3 + (M_{ij} + M_{ji})\,\beta_4 + X_{ij}\,\beta_5 + \epsilon_{iji} \qquad (1)$$

$$T_{ij} = \gamma_0 + S_{ij}\gamma_1 + X_i\gamma_2 + X_j\gamma_3 + M_{ij}\gamma_4 + X_{ij}\gamma_5 + \epsilon_{ij} \qquad (2)$$

$$\frac{O_{ijt}}{O_{ij}} = \frac{[\,(\lambda \exp(Z_{ijt}\,\theta) + 1)^{1/\lambda} + \mu]\,\exp(u_{ijt})}{\sum_{v=1}^{n_{ij}} [\,(\lambda \exp(Z_{ijv}\,\theta) + 1)^{1/\lambda} + \mu]\,\exp(u_{ijv})}, \qquad (3)$$

$$S_{ij} \equiv \log\{\sum_{v=1}^{n_{ij}} \exp(Z_{ijv}\,\theta)\}, \qquad (4)$$

---

[2] To ease the exposition, the term "travel" replaces the usual expressions circle and open-jaw.

The general form of the round trip equation corresponds to the so-called "direct demand by mode" proposed by Kraft and Wohl (1967). This form allows the following interpretations for the socioeconomic variables associated to a round trip iji: the vectors X_i and X_j denote respectively the number of trips generated from city i and the number of trips attracted to city j.

In order to explain the total air travel demand, we included the one-way trip equation. The socioeconomic variables in (2) have a different interpretation from (1); it cannot be determined whether the generation of the trip should be attributed to i or to j. The reason lies in Long's citation mentioned in the introduction.

As for most air travel demand models, we retained the log-linear functional form for (1) and (2).

The share model (3) corresponds to the IPT-L model introduced by Gaudry (1978). An interesting special case of (3) is the logit model when $\lambda = 1$ and $\mu = -1$. The IPT-L model allows, contrary to the logit model, the introduction of a unique constant within the vector Z_{ijt} for all alternatives. The relevance of the constant in the share model will be discussed in the section 2.3 . The variable S_{ij}, which is the level of service for the air mode in market ij, is the familiar inclusive value for a choice set when the share model is the logit (Mcfadden, 1978)[3]. Equation (3) gives meaning to the variable S_{ij} and computes the share of a new itinerary in a market.

The aggregation of various characteristics of air trips proceeds from two steps: the first being the definition of an itinerary, i.e. a set of paths with similar values for some characteristics, namely: travel time, transfer time and fare. Thus we distinguish the paths with the dimensions most likely to influence total demand. Of course, the similitude of the paths contained in the itineraries could be increased by adding characteristics to define the itineraries but, since our main objective is to show the relevance of disaggregating the trips, we believe that the selected criteria are satisfactory. The second step is given by the S_{ij} variable. A complete description of the aggregation will be presented in a forthcoming article.

One could interpret the equation system (1) to (4) as an econometric model to estimate a set of differentiated demands. Indeed the equations (1) and (2) represent the explanation of an aggregated measure of the total demand and the equation (3) being the relative share of each differentiated good according to its characteristics and those of the competing goods. In the past, other authors (McLynn and Woronka, 1969; Monsod, 1966; Gaudry and Wills, 1978) have used a total demand and share model together but they never estimated them simultaneously nor was it applied for the aggregation of the paths.

---

[3] The inclusive value is defined as
$E\{\max(Z_{ijt}\theta + u_{ijt})\}$ if u_{ijt} is identically independently Weibull distributed.

To evaluate the numbers T_{iji} and T_{ij}, we created a unique data bank from the audit coupons (about 12,000,000) used by Air Canada and CP Air during the year 1983. This data bank represents approximately 65% of the Canadian air travel demand. Both the confidentiality and the unavailability of similar data from other carriers have limited the carrier number in our sample. The specification of the demand model takes this fact partially into account.

The following information appears on each coupon received from the carriers: ticket number, coupon number, traveling date, carrier code, flight number, fare, fare class, departure and arrival cities. With the flight number and the traveling date we can retrieve from the Official Airline Guide North American Edition (1983) the arrival and departure times at the cities appearing on the coupons.

Two rules enable the gathering of coupons related to a same ticket:
i) all the coupons (at most four) in a ticket have the same ticket number;
ii) if a travel requires many tickets, the tickets have consecutive numbers.

We classified the travels into two groups: complete and incomplete.[4] For a completed travel we associated the travelers' place of residence to the first city of the travel. The validity of this hypothesis is generally verified, except occasionaly for the cities of Montreal and Toronto where, in order to benefit from more advantageous schedules, residents from other cities of the sample travel by other modes to Montreal or Toronto.

1.2 SPECIAL CASES

In this section, we first derive and discuss the ANITA trip model which generalizes all trip models found in literature and gives new meaning to network and socioeconomic variables. We recall that this is so because of the disaggregation of the usual trip data into complete and incomplete travels. Then we propose a classification of the air trip demand models found in literature and show the restrictions under which they appear as special cases of the ANITA trip model.

Statistics Canada (S.C.) and the Official Airline Guide (O.A.G.) in the U.S. publish the number of trips by plane from city i to city j during one year (O_{ij}). We propose to disaggregate the trips with:
i) trips ij contained in round trips T_{iji};
ii) trips ij contained in round trips T_{jij};
iii) one-way trips from i to j, T_{ij}, i.e.

$$O_{ij} = T_{iji} + T_{jij} + T_{ij}. \qquad (5)$$

[4] An incomplete travel may occur in event of multi-modal travels, travels started in 1982 or completed in 1984, the purchase of coupons for trips only, and travels completed with carriers not included in our sample.

The ANITA trip model (6) is obtained by substituting (1) and (2) in (5).

$$O_{ij} = \gamma_0 + (S_{ij} + S_{ji})\,\gamma_1 + S_{ij}\,\gamma_2 + X_{ij}\gamma_3 + X_i\,\gamma_4 + X_j\,\gamma_5 + (M_{ij} + M_{ji})\,\gamma_6 + M_{ij}\,\gamma_7 + \xi_{ij}, \quad (6)$$

where

$$\gamma_0 = 2\,\beta_0 + \gamma_0,\ \gamma_1 = 2\,\beta_1,\ \gamma_2 = \gamma_1,$$
$$\gamma_3 = 2\,\beta_5 + \gamma_5,\ \gamma_4 = \beta_2 + \beta_3 + \gamma_2,\ \gamma_5 = \beta_2 + \beta_3 + \gamma_3$$
$$\gamma_6 = 2\,\beta_4,\ \gamma_7 = \gamma_4,\ \xi_{ij} = \epsilon_{iji} + \epsilon_{jij} + \epsilon_{ij}. \quad (7)$$

Two characteristics distinguish the ANITA trip model from other proposed trip models:

- generalized directional effect of the network variables (GDN);
- generalized directional effect of the socio-economic variables (GDS).

Generalized directional effect of the network variables (GDN)

Network variables have directional effects for they induce, caetirus paribus, a directional number of trips ($O_{ij} \neq O_{ji}$), i.e.:

$$(S_{ij} + S_{ji})\,\gamma_1 + S_{ij}\,\gamma_2 \neq (S_{ji} + S_{ij})\,\gamma_1 + S_{ji}\,\gamma_2, \text{ and}$$
$$(M_{ij} + M_{ji})\,\gamma_6 + M_{ij}\,\gamma_7 \neq (M_{ji} + M_{ij})\,\gamma_6 + M_{ji}\,\gamma_7, \quad (8)$$

This is so since the network variables are asymmetrical, i.e. they have different values for markets ij and ji ($S_{ij} \neq S_{ji}$, $M_{ij} \neq M_{ji}$).

Furthermore, the effect of the network variables is generalized, since trips ij in the equation (6) are explained not only by the level of service for the market ij but also, and distinctively, by the level of service for the market ji, i.e.

$$S_{ij}\,(\gamma_1 + \gamma_2) \neq S_{ji}\,\gamma_2, \text{ and}$$
$$M_{ij}\,(\gamma_6 + \gamma_7) \neq M_{ji}\,\gamma_7 \quad (9)$$

Even if $S_{ij} = S_{ji}$, the γ parameters cause a generalized character effect on the network variables. This is not so for the directional character.

Generalized directional effect of the socioeconomic variables (GDS)

A socioeconomic variable (x_{ik})[5] has directional effects for it produces, caetirus paribus, a directional number of trips ($O_{ij} \neq O_{ji}$), i.e.

$$x_{ik}\,\gamma_4 \neq x_{ik}\,\gamma_5. \quad (10)[6]$$

Thus, the condition for a directional effect becomes $\gamma_3 \neq \gamma_4$.

The socioeconomic variables have generalized effect due to the identification of the effect associated to the residents of i (β_{2k}, β_{3k}), the effect related to the travelers stemming from i (γ_{2k}) and to the travelers going to j (γ_{3k}).

---

[5] This is an element of the vector X_i.

[6] For the observation ij the x_{ik} variable is associated to the origin (γ_4 parameter) whereas for the observation ji it is related to the destination (γ_5 parameter).

We now define four categories of constraints for the equation (6) in order to derive the trip models proposed in literature. These categories are:

- nondirectional effects of the network variables (NN);
- particular directional effects of the network variables (PDN);
- nondirectional effects of the socioeconomic variables (NS);
- particular directional effects of the socioeconomic variables (PDS).

Nondirectional effects of the network variables (NN)

In (8) if $\gamma_2 = \gamma_7 = 0$ ($\gamma_1 = \gamma_4 = 0$) then the network variables produce nondirectional number of trips. In some studies (CTC, 1976; Lansig, 1961; Brown and Watkins, 1968), the distance or the fare is a symmetrical network variable and in others (De Vany, 1974; Howrey, 1969) the frequency or the travel time for direct flights is presumed symmetrical. Nondirectional effect implies a recognition of the characteristics for the trips of the opposite direction.

Particular directional effects of the network variables (PDN)

Usually, the network variables are not symmetrical, i.e. $S_{ij} \neq S_{ji}$, $M_{ij} \neq M_{ji}$, but the network variables for market ji do not appear in the list of regressors for trips ij.

The constraints $\gamma_1 = \gamma_6 = 0$ ($\beta_1 = \beta_4 = 0$) imply a directional effect for the network variables and withdraw the network variables of the opposite direction.

Nondirectional effects of the socioeconomic variables (NS)

Many trip models imply the same effect on the socioeconomic variables (X_i) associated with the origin and those associated with the destination (X_j).[7] We obtain this formulation by imposing $\gamma_4 = \gamma_5$ ($\gamma_2 = \gamma_3$) in (6).

This formulation can be found in: Abrahams (1980), Andrikopoulos (1980), Brown and Watkins (1968), De Vany (1974), Howrey (1969), Ippolito (1981), Olson and Trapani (1981), Oum et al.(1984), CTC (1976), Sobienak (1972).

Particular directional effects of the socioeconomic variables (PDS)

For the cases other than $\gamma_2 = \gamma_3$, the effect of socioeconomic variables is necessarily directional, but one can only identify the vector of parameters $\beta_2+\beta_3+\gamma_2$ associated to X_i and $\beta_2+\beta_3+\gamma_3$ associated to X_j.

In conclusion, the one-way trip equation affects the directional character of the explanatory variables. Indeed, if the γ's are equal to zero for the network variables or the socioeconomic variables are equal, then the explanatory variables have no directional effect.

Here are the general forms of the ANITA trip model (6) and the proposed trip models, (11)-(14).

[7] The invoked reason is found in the footnote mentioned in the introduction.

$$O_{ij} = f\ (S_{ij} + S_{ji},\ S_{ij},\ M_{ij} + M_{ji},\ M_{ij},\ X_i,\ X_j,\ \ ,\ X_{ij}) \qquad (6)'$$

(NN, PDS) ($\Psi_2 = \Psi_7 = 0$)

$$= f\ (S_{ij} + S_{ji},\ \ ,\ M_{ij} + M_{ji},\ \ ,\ X_i,\ X_j,\ \ ,\ X_{ij}) \qquad (11)$$

Lansig (1961)

(NN, NS) ($\Psi_2 = \Psi_7 = 0,\ \Psi_4 = \Psi_5$)

$$= f\ (S_{ij} + S_{ji},\ \ ,\ M_{ij} + M_{ji},\ \ ,\ \ ,\ \ ,\ X_i + X_j,\ X_{ij}) \qquad (12)$$

Brown and Watkins (1968), De Vany (1974), Howrey (1969), Rae (1976)

(PDN, NS) ($\Psi_1 = \Psi_6 = 0,\ \Psi_4 = \Psi_5$)

$$= f\ (\ \ ,\ S_{ij},\ \ ,\ M_{ij},\ \ ,\ \ ,\ X_i + X_j,\ X_{ij}) \qquad (13)$$

Abrahams (1980), Andrikopoulos (1981), Ippolito (1981), Olson (1981), Oum et al.(1984), Sobieniak (1972), Trapani (1981)

(PDN, PDS) ($\Psi_1 = \Psi_6 = 0$)

$$= f\ (\ \ ,\ S_{ij},\ \ ,\ M_{ij},\ X_i,\ X_j,\ \ ,\ X_{ij}) \qquad (14)$$

Darrow (1978), Gronau (1970), Gulley (1970), Long (1968,1969), Straszheim (1978), Trapani (1982), Verlerger (1972)

Because of (5) and its capacity to represent data associated to a trip, we propose to compare our general approach, i.e. round trips and one-way trips, with the usual approach, namely trips. However, the validity of (5) is weakened by three facts:

1) the completed travels with more than two trips, including trips ij, are missing from the definition. We omitted them since they represent less than 3% of the trips in our sample and increase considerably the complexity of the analysis[9];
2) the selection of destinations in the travels includes planned sojourns exceeding 2½ hours[9], which is different from the criteria "the farther city from the first city of the travel" used by S.C. and O.A.G.;
3) instead of logarithmic values,[10] a strict definition would necessitate the untransformed variables on the right hand of (5). A high correlation (.93) between the trips defined by (5) and the trips with untransformed T_{ij} and T_{ji} dictated our choice. It should be noted that for a linear trip model this distinction is not relevant.

Thus, (5) should be perceived as an excellent representation, albeit an imperfect one, of the trips published by S.C. and O.A.G. However, we believe that none of these discrepancies significantly reduces the interest of the comparisons based on (5).

---

[9] For similar reasons Pinton(1967) did not treat the trips with many goals.
[9] This criteria was selected after discussions with the airlines.
[10] We already mentioned that we selected the log-linear functional form.

1.3 ESTIMATION PROCEDURE

In order to stipulate the advantages of recognizing the travelers' place of residence, we shall use the following procedure:

i) estimate the ANITA model, i.e. (1), (2), and (3) without constraints;
ii) implement a set of constraints for the Ψ's of (6) and estimate with the sample of trips, such as defined by (5), the remaining parameters Ψ along with the θ's of (3);
iii) estimate again (1) and (2) subject to the θ's and Ψ's computed in ii);
iv) compute the differences between the maximum value of the log-likelihood functions for i) and iii).

From the samples of one-way trips and round trips, we defined a general model. Then, we compared this general model to particular trip models parametrized with the trip sample.

Step i) is self-explanatory. Step ii) concerns the choice and the estimation of one particular trip model: NR-NS, NR-DPS, DPR-NS, DPR-DPS.

In step iii) we compute the maximum log-likelihood value for (1), (2), and (3), conditional to the parameters computed in the second step. In order to do so, we proceed as follows:

$$T_{iji} = \beta_0 + (\hat{S}_{ij} + \hat{S}_{ji})\,\hat{\Psi}_1/2 + X_i\,\beta_2 + X_j\,\beta_3 + (M_{ij} + M_{ji})\,\hat{\Psi}_6/2 + X_{ij}\,\beta_5 + \epsilon_{iji} \qquad (15)$$

$$T_{ij} = \hat{\Psi}_0 - 2\beta_0 + \hat{S}_{ij}\,\hat{\Psi}_2 + X_i\,(\hat{\Psi}_4 - \beta_2 - \beta_3) + X_j\,(\hat{\Psi}_5 - \beta_2 - \beta_3) + M_{ij}\,\hat{\Psi}_7 + X_{ij}\,(\hat{\Psi}_3 - 2\beta_5) + \epsilon_{ij}. \qquad (16)^{11}$$

The parameters with the symbol "^" in (15) and (16) are computed or fixed in step ii). Let us define the following dependent variables,

$$T^*_{iji} \equiv T_{iji} - (\hat{S}_{ij} + \hat{S}_{ji})\hat{\Psi}_1/2 - (M_{ij} + M_{ji})\hat{\Psi}_6/2, \qquad (17)$$

$$T^*_{ij} \equiv T_{ij} - \hat{\Psi}_0 - \hat{S}_{ij}\hat{\Psi}_2 - X_i\hat{\Psi}_4 - X_j\hat{\Psi}_5 - M_{ij}\hat{\Psi}_7 - X_{ij}\hat{\Psi}_3. \qquad (18)$$

Thus (15) and (16) become:

$$T^*_{iji} = \beta_0 + X_i\,\beta_2 + X_j\,\beta_3 + X_{ij}\,\beta_5 + \epsilon_{iji}, \qquad (19)$$

$$T^*_{ij} = -2\beta_0 + (-X_i - X_j)\,\beta_2 + (-X_i - X_j)\,\beta_3 + (-2X_{ij})\,\beta_5 + \epsilon_{ij}. \qquad (20)$$

Equations (19) and (20) include the constraints given by the Ψ's estimated in step ii). To fully define the likelihood function, we must add the constraints given by (21) and (22):

$$\hat{\sigma}^2_{\xi_{ij}} = 2\sigma^2_{\epsilon_{iji}} + \sigma^2_{\epsilon_{ij}} \qquad (21)$$

$$\theta = \hat{\theta}_{od}. \qquad (22)$$

The likelihood function and its derivatives are given in Appendix 2.

---

[11] The symbol "^" over the S_{ij} variables means that the θ parameter is fixed to the values obtained in step ii).

PART 2
RESULTS

2.1 ELASTICITIES

Table 1 contains the elasticities and the *t*-statistics for the different formulations. The computation of the *t*-statistics is explained in Appendix 2. The elasticities for the air network variables ($Z_{ij,t}$) are computed with (27).

$$\eta(\bar{T}, \bar{Z}_k) \equiv \frac{\partial \bar{T}}{\partial Z_k} \frac{\bar{Z}_k}{\bar{T}} = \delta_1 \theta_k \bar{Z}_k / \bar{n} \qquad (27)$$

where Z_k is a particular variable of the vector Z, $\bar{n}$ is the average number of itineraries in the market, T is the untransformed round trips or one-way trips,[12] δ replaces β or γ. The elasticities are computed with the average value of the positive observations for these variables.[13] The elasticities of the other variables (X_i, X_j, M_{ij}, X_{ij}) in (1) and (2) are expressed by (28)

$$\frac{\partial \bar{T}}{\partial \bar{Z}_k} \frac{\bar{Z}_k}{\bar{T}} = \begin{cases} \cdot\ \delta_k \bar{Z}_k, & \text{if } Z_k \text{ is a dummy or quasi-dummy variable,} \\ \cdot\ \delta_k, & \text{otherwise} \end{cases} \qquad (28)^{14}$$

We recall that the elasticities in Table 1 concern only the demand of Air Canada and CP Air. For this reason, we will not compare the elasticities from Table 1 with those obtained from other authors. Furthermore, a modification of the definitions of elasticity would be essential. Sensibility should be considered for trips rather than for round or one-way trips.

2.2 INVERSE POWER TRANSFORMATION BINOMIAL LOGIT PARAMETERS

Parameters λ and μ in (3) are fixed respectively to 1.1 and -.99. The values of these parameters are close enough to the logit values so as to allow the estimation of the constant (see section 2.1) and preserve the meaning of S_{ij}, the level of service variable. The estimation of λ and μ would have necessitated the introduction of the I.P.T-L parameters in the definition of S_{ij}.[15]

---

[12] One should remember that $T_{ij,t}$ and T_{ij} in the equations (1) and (2) are transformed with the logarithmic function.

[13] This definition corresponds to equation (43) in Dagenais et al. (1987).

[14] The code of the quasi dummies and the dummies are underlined in table 1.

[15] We intend to include the parameters of the I.P.T. model in the level of service variable in subsequent work but felt it was unnecessary for our needs.

TABLE 1: ROUND TRIPS

VARIABLE	CODE	1)GDN-GDS	2)NN-PDS	3)NN-NS	4)PDN-NS	5)PDN-PDS
Air network						
travel time	TRAVELT	-1.143	-2.125	-2.135	0.000	0.000
		(-14.702)	(-17.543)	(-17.571)	(---)	(---)
transfer time	TRANSFET	-1.260	-1.864	-1.870	0.000	0.000
		(-21.102)	(-18.662)	(-18.646)	(---)	(---)
price of the trip	COST	-1.145	-1.476	-1.480	0.000	0.000
		(-10.688)	(-8.899)	(-8.893)	(---)	(---)
number of departures	FREQUEN	.283	.529	.532	0.000	0.000
		(22.038)	(17.658)	(17.695)	(---)	(---)
first class	FIRST C	.566	.558	.562	0.000	0.000
		(9.168)	(4.455)	(4.471)	(---)	(---)
second class	SECND C	.711	1.350	1.363	0.000	0.000
		(12.644)	(13.636)	(13.704)	(---)	(---)
third class	THIRD C	.189	.321	.325	0.000	0.000
		(3.209)	(3.348)	(3.369)	(---)	(---)
two itineraries	CTE1	1.082	2.036	2.042	0.000	0.000
		(13.626)	(15.770)	(15.745)	(---)	(---)
2+ itineraries	CTE2	.788	1.666	1.666	0.000	0.000
		(9.497)	(11.722)	(11.668)	(---)	(---)
level of service	SIJ +SJI	.356	.577	.579	0.000	0.000
		(33.242)	(.763)	(.762)	(---)	(---)
Network of the substitute modes						
time car, bus	TB*TC	.081	.007	.004	0.000	0.000
		(1.476)	(.002)	(.001)	(---)	(---)
number of departures	FBUS	-0.124	-.656	-.653	0.000	0.000
		(-2.578)	(-.200)	(-.199)	(---)	(---)
bus fare	$BUS	.664	2.155	2.172	0.000	0.000
		(6.092)	(.207)	(.209)	(---)	(---)
nights	NIGHTCAR	.067	-.108	-.115	0.000	0.000
		(1.116)	(-.023)	(-.025)	(---)	(---)
frequencies (others)	FCOMP	-0.110	-.122	-.121	0.000	0.000
		(-4.170)	(-.079)	(-.078)	(---)	(---)
air distance	DP	-0.532	-1.054	-1.046	-1.149	-1.150
		(-6.450)	(-.141)	(-.140)	(-3.298)	(-3.309)
boat	DBOAT	-0.268	-.270	-.268	-.005	-.005
		(-3.596)	(-.047)	(-.047)	(-.015)	(-.014)
Socioeconomic variables						
attractiveness	POPGRAV OD	.111	.044	.047	-.186	-.186
		(2.474)	(.014)	(.015)	(-1.021)	(-1.025)
linguistic affinity	CULTURE OD	.239	.422	.421	.610	.611
		(7.281)	(.185)	(.185)	(3.421)	(3.433)
in-migrants (prov)	MIGIAPR O	.115	.148	.150	-.047	-.047
		(3.249)	(.058)	(.058)	(-.315)	(-.318)
population of origin	POP83 O	.266	.285	.281	.486	.487
		(7.764)	(.111)	(.110)	(3.542)	(3.554)
university of origin	UNIV O	.160	.193	.191	.483	.484
		(6.238)	(.098)	(.097)	(5.116)	(5.137)
income of origin	INCOME O	2.040	2.555	2.550	4.942	4.944
		(6.695)	(.095)	(.095)	(3.754)	(3.766)
constant	CONSTANT	-48.860	-67.705	-67.607	-109.418	-109.461
		(-12.565)	(-.204)	(-.203)	(-6.362)	(-6.378)
out-migrants (prov)	EMIAPR D	.166	-.053	-.055	.152	.152
		(2.693)	(-.012)	(-.012)	(.609)	(.612)
population of dest.	POP83 D	.101	.344	.340	.510	.510
		(2.062)	(.095)	(.094)	(2.813)	(2.823)
university of dest.	UNIV D	.179	.271	.271	.397	.397
		(7.362)	(.139)	(.139)	(4.085)	(4.100)
income of dest.	INCOME D	1.933	2.747	2.748	4.433	4.434
		(7.710)	(.137)	(.137)	(4.149)	(4.160)
MULTIPLE CORRELATION	R^2	.835	.249	.272	.272	.249
LOG-LIKELIHOOD FUNCTION	$\mathcal{L}$	-1357.543	-2254.191	-2256.557	-2715.210	-2715.045
degrees of freedom		29	29	29	29	
χ^2 COMPUTED		1793.30	1798.03	2715.33	2715.01	
χ^2 TABULATED (5%)		42.56	42.56	42.56	42.56	
SAMPLE SIZE		766	766	766	766	

TABLE 1: ONE-WAY TRIPS (cont'd)

VARIABLE	CODE	1)GDN-GDS	2)NN-PDS	3)NN-NS	4)PDN-NS	5)PDN-PDS
Air network						
travel time	TRAVELT	-1.062	0.000	0.000	-3.826	-3.804
		(-14.702)	(---)	(---)	(-16.288)	(-16.300)
transfer time	TRANSFET	-1.170	0.000	0.000	-3.504	-3.481
		(-21.102)	(---)	(---)	(-18.461)	(-18.508)
price of the trip	COST	-1.064	0.000	0.000	-2.880	-2.856
		(-10.688)	(---)	(---)	(-9.010)	(-9.004)
number of departures	FREQUEN	.263	0.000	0.000	1.050	1.041
		(22.038)	(---)	(---)	(20.754)	(20.706)
first class	FIRST C	.526	0.000	0.000	1.065	1.049
		(9.168)	(---)	(---)	(4.729)	(4.698)
second class	SECND C	.660	0.000	0.000	2.428	2.401
		(12.644)	(---)	(---)	(13.121)	(13.084)
third class	THIRD C	.175	0.000	0.000	.559	.556
		(3.209)	(---)	(---)	(3.086)	(3.090)
two itineraries	CTE1	1.006	0.000	0.000	3.394	3.373
		(13.626)	(---)	(---)	(14.047)	(14.057)
2+ itineraries	CTE2	.732	0.000	0.000	2.280	2.320
		(9.497	(---)	(---)	(8.608)	(8.821)
level of service	SIJ +SJ1	.661	0.000	0.000	2.143	2.128
		(31.516)	(---)	(---)	(.405)	(.418)
Network of the substitute modes						
time car, bus	TB*TC	.162	0.000	0.000	.235	.232
		(2.617)	(---)	(---)	(.033)	(.034)
number of departures	FBUS	-.153	0.000	0.000	-.763	-.826
		(-4.456)	(---)	(---)	(-.148)	(-.164)
bus fare	$BUS	.588	0.000	0.000	3.193	3.260
		(5.824)	(---)	(---)	(.191)	(.200)
nights	NIGHTCAR	.076	0.000	0.000	-.388	-.393
		(1.253)	(---)	(---)	(-.048)	(-.050)
frequencies (others)	FCOMP	-.051	0.000	0.000	-.096	-.102
		(-1.624)	(---)	(---)	(-.039)	(-.042)
air distance	DP	-.755	-1.295	-1.227	-.570	-.682
		(-7.733)	(-5.615)	(-5.265)	(-.060)	(-.075)
boat	DBOAT	-.095	.260	.226	-.218	-.163
		(-1.263)	(1.081)	(.945)	(-.025)	(-.019)
Socioeconomic variables						
attractiveness	POPGRAV OD	.133	-.245	-.165	.269	.241
		(3.062)	(-1.855)	(-1.265)	(.048)	(.045)
linguistic affinity	CULTURE OD	.306	.326	.345	.086	.034
		(10.280)	(2.875)	(3.039)	(.021)	(.008)
in-migrants (prov.)	MIGIAPR O	.116	.363	.392	.448	.350
		(3.622)	(3.341)	(3.554)	(.102)	(.085)
population of origin	POP83 O	.250	.114	.171	-.248	-.226
		(7.258)	(1.133)	(1.681)	(-.053)	(-.050)
university of origin	UNIV O	.202	.303	.233	-.064	.080
		(8.617)	(4.349)	(3.362)	(-.019)	(.025)
income of origin	INCOME O	2.215	2.739	3.273	-.371	-1.488
		(7.326)	(2.877)	(3.445)	(-.009)	(-.037)
constant	CONSTANT	-59.706	-78.701	-77.027	.158	-2.092
		(-15.944)	(-7.010)	(-6.779)	(.000)	(-.004)
out-migrants (prov)	EMIAPR D	.295	.831	.648	.352	.885
		(5.220)	(4.656)	(3.580)	(.050)	(.128)
population of dest.	POP83 D	.092	.127	.171	-.248	-.483
		(2.116)	(.969)	(1.273)	(-.042)	(-.081)
university of dest.	UNIV D	.127	.128	.233	-.064	-.308
		(5.866)	(2.012)	(3.530)	(-.021)	(-.098)
income of dest.	INCOME D	2.913	3.989	3.273	-.371	.887
		(12.429)	(6.105)	(4.855)	(-.011)	(.029)
MULTIPLE CORRELATION	R^2	.848	.234	.234	.185	.185
LOG-LIKELIHOOD FUNCTION	$\mathcal{L}$	-1357.543	-2254.191	-2256.557	-2715.210	-2715.045
degrees of freedom			29	29	29	29
χ^2 COMPUTED			1793.30	1798.03	2715.33	2715.01
χ^2 TABULATED (5%)			42.56	42.56	42.56	42.56
SAMPLE SIZE			957	957	957	957

2.3 ANALYSES OF THE RESULTS FOR THE ANITA MODEL

Before evaluating the advantages of the recognition of the travelers' place of residence, we analyzed the results obtained from the ANITA (GDN-GDS) model. The description of the variables listed in Table 1 is found in Appendix 1; for more details, see Laferrière (1988). Unless otherwise specified, the comments pertain to round and one-way trip equations.

Air network

We expect a greater elasticity of the first and regular economy class demands, with respect to the time rather than to the fare, and the opposite for the other fare class demands. The grouping of the fare classes may have affected the similitudes of the demand elasticities with respect to TRAVELT and COST. As expected, but at a smaller extent, TRAVELT implies a greater utility than TRANSFET.

The demands are more sensitive, caeteris paribus, to the presence of first and second classes rather than to the presence of the third class. The service on board may explain this result, but FIRST C and SECOND C could also reflect the presence of important markets (ex.: Toronto-Montreal, Toronto-Ottawa, etc.).

The fact that CTE1 is greater than CTE2 in the two equations suggest that the network variables have a greater share of the explanation for markets with more than 2 itineraries. It also means that the level of the demand is less for markets with more than 2 itineraries than those with just 2 itineraries. One interpretation of this could be that many itineraries occur for markets with substantial differences in the waiting time, i.e. paths differentiating because of various stopovers.

Network of the substitute modes

All the variables in this category have the expected sign, except maybe for DBOAT. A positive sign may denote an increase of discomfort with terrestrial mode, but a negative sign may signify less exchange than expected between the city pairs, as a result of the necessity to take the ferry when traveling by car or by bus between the cities. Note that the elasticities for TB*TC, FBUS, $BUS and FCOMP have, as anticipated, a smaller magnitude than their counterpart in the air network category.

The presence of DP and the equality of TB and TC coefficients reflect a specification problem. This problem has been seen by others (Domenchich et al. 1968; Peers and Bevilaqua, 1976). Although recent research (Gaudry and Dagenais, 1987) has shed a light in that field, there is more research to be done in order to solve this particular problem for unimodal direct demand model.

Socioeconomic variables

Among the characteristics of air travelers, personal income is certainly a distinctive one (Darwin, 1982). This seems to be corroborated with RMPMOY O and RMPMOY D. For the round trip equation, the only variables having different effects when used in origin or destination are POP83 O and POP83 D, whereas for the one-way trip equation the effects of all the variables differ.

2.4 GAINS FROM THE RECOGNITION OF THE TRAVELERS' PLACE OF RESIDENCE

Columns 2 to 5 in Table 1 present the elasticities obtained from four conventional formulations. See equations (15), (16), (17), and (18) for a schematic presentation of the proposed trip model formulation. We disposed the trip models according to their occurences in literature: from the seldom used NN-PDS to the relatively popular PDN-PDS.

From Table 1 we can observe that:

i) the network variables are more appropriate in the round trip equation than in the one-way trip equation. This statement comes from comparing ξ in the NN formulation (columns 2 and 3) and in the PDN formulation (columns 4 and 5);

ii) the PDS formulation (columns 2 and 5) shows little difference when compared with the NS formulation (columns 3 and 4);

iii) the conventional formulations produce many sign inversions for the socioeconomic variables. The one-way trip equation with the PDN specification affects the majority of them;

iv) all the conventional specifications are rejected according to the likelihood ratio test. Indeed, all the computed χ^2 statistics greatly exceed the tabulated χ^2 at a significance level of 5% (42.56).

The following conclusions can be drawn from the preceding remarks:

i) the trip characteristics for the opposite directions (S_{ji}, M_{ji}) should be included as explanatory variables. For the round and one-way trip samples, better results are obtained using bi-directional network variables. Therefore, by definition, the trips are clearly explained using network variables for the two directions;

ii) contrary to the network variables, the distinction between directional and non-directional effects for the socioeconomic variables is not supported by the data. This may explain the popularity of the two formulations;

iii) the numerous sign inversions arising from the PDN specification in the one-way trip equation suggest a substantial influence of the network specification upon the socioeconomic variables. More precisely, the dissimilarities and similitudes of columns 2) to 5) in Table 1 are linked to the network variable formulation, i.e. NN-PDS is similar to NN-NS as PDN-NS is to PDN-PDS, but NN-PDS and NN-NS differ from PDN-NS and PDN-PDS;

iv) the estimated coefficients with the trips' data ($Q_{i,j}$) in the models NN-PDS, NN-NS, PDN-NS, and PDN-PDS lead to a poor explanation of the round trips ($T_{i,j,i}$) and the one-way trips ($T_{i,j}$) which imply that *t*-statistics and squared multiple correlation coefficients are much smaller than those in column 1;

v) there are very important differences between the results of the ANITA model (GDN-GDS) and those of the conventional trip model. The elasticities and the log-likelihood function reflect those dissimilarities. The recognition of the travelers' place of residence seems more than appropriate and must be compared to the network variables specification.

CONCLUSION

This article has studied the benefits for the travel demand estimation to disaggregate trips such as those published by statistical offices in North America. The disaggregation has allowed the formulation of a generalized trip demand model and the recognition of the travelers' place of residence.

The recognition of the travelers' place of residence brings forth major differences for the air travel demand. These differences are reflected by considerable changes in the elasticities and also by meaningful higher values for the maximum likelihood function. An interesting specification for the round and one-way trips becomes quite unsatisfactory when estimated with a trip sample. All conventional trip models (NN-P , NN-NS, PDN-NS, PDN-PDS, GDN-GDS) have many sign inversions.

The advantages of knowing and taking into account the travelers' place of residence seem to be greater in network variables than in socioeconomic variables.

The demand model developed in this article could not at this present stage of development be used for other goals because of several shortcomings: the information for all the major airlines would then be required, the exogeneity of the price variable could no longer be assumed as it was at the time of this study and the specification of the demand model would have to be elaborated.

REFERENCES

ABRAHAMS, M.B., **A Simultaneous Equation Estimation of Air Travel Demand**, Ph.D. thesis, University of California, 1980, 91 p.

ANDRIKOPOULOS, A.A. and T. BAXEVANIDIS, "Interurban Demand for Air-travel Services: The Case of Canada", **International Journal of Transport Economics**, vol. VIII (3), 1981.

BERNDT, E.K., B.H. HALL, R.E. HALL and J.A. HAUSMAN, "Estimation and Inference in Nonlinear Structural Models", **Annals of Economic and Social Measurement**, 3, 1974.

BROWN, S.L. and S. WATKINS, "The Demand for Air Travel: A Regression Study of Time-Series and Cross-Sectional Data in the U.S. Domestic Market", **Highway Research Record**, 213, 1968.

DARROW, D.F. and F.R. WILSON, "Air Travel Demand for Atlantic Canada", **Tribune de l'ARTC**, vol. 2 (4).

DARWIN, E., "Canadian Intercity Passenger Transportation: An Overview", Transport Canada, TP3760, 1982.

DE VANY, A., "The Revealed Value of Time in Air Travel", **Review of Economics and Statistics**, vol. 66 (1), 1974.

DE VANY, A., "The Effect of Price and Entry Regulation on Airline Output, Capacity, and Efficiency, **Bell Journal of Economics**, 6, 1975.

DOMENCHICH T.A., G. KRAFT and J.-P. VALETTE, "Estimation of Urban Travel Behavior: An Economic Demand Model", in **Transportation System Evaluation.** Highway Research Record 238, Washington, 1968.

GAUDRY, M.J.I., "The Inverse Power Transformation Logit and Dogit Mode Choice Models", **Transportation Research** B 15, 2, 1981.

GAUDRY, M.J.I. and M. DAGENAIS, **Can Aggregate Demand Work?**, Proceedings of the World Conference on Transportation Research, 1986, Center for Transportation Studies, University of British Columbia, p. 1669-1676.

GAUDRY, M.J.I. and M.J. WILLS, "Estimating the functional form of travel demand models", **Transportation Research** 12, 1978.

GRONAU, R., **The Value of Travel Time. Theory and Measurement**, Croom Helm, London, 1970, 213 p.

GULLEY, E., "Forecasting Intercity Travel", **Transportation Research Forum**, 1970.

HOWREY, E.P., "On the Choice of Forecasting Models for Air Travel", **Journal of Regional Science**, vol. 9 no. 2, 1969, pp. 215-224.

IPPOLITO, R.A., "Estimating Airline Demand with Quality of Service Variables", **Journal of Transportation Economics and Policy**, 1981, pp. 7-14.

KRAFT, G. and M. WOHL, "New Directions for Passenger Demand Analysis and Forecasting", **Transportation Research**, vol. 1, 1967, pp. 205-230.

LAFERRIERE, R., "Une Aggrégation Nouvelle des Itinéraires de Transport Aérien (ANITA)", Ph.D. thesis, Centre de Recherche sur les Transports, publication 574, 1988.

LANSIG, J.B., J.-C. LAO and D.B. SUITS, "An Analysis of Interurban Air Travel", **Quarterly Journal of Economics**, 1961, pp. 87-95.

LONG, H.W., "City Characteristics and the Demand for Interurban Air Travel", **Land Economics**, 1968.

LONG, H.W., "Airline Service and the Demand for Intercity Air Travel", **Journal of Transport Economics and Policy**, 3, 1969, pp. 287-299.

MONSOD, C., "A cross-sectional model of the demand for rail passenger service in the Northeast Corridor", **Studies in Travel Demand**, vol. II, Mathematica, 1966, pp. 157-178.

McLYNN, J.M. and T. WORONKA, "Passenger Demand and Modal Split Models", Northeast Corridor Project, U.S. Department of Transportation, Report #230, 1969.

OLSON, C.V. and J.M. TRAPANI III, "Who Has Benefited From Regulation of the Airline Industry", **Journal of Law and Economics**, 26, 1981, pp. 75-93.

OUM, T.H., D.W. GILLEN and D.S.E. NOBLE, **Demand for Fare Classes and Pricing in Airline Markets**, Research Report #8472, Wilfrid Laurier University, 1984, 36 p.

PEERS, J.B. and M. BEVILAQUA, "Structural Travel Demand Models: An Intercity Application", **Transportation Research Board**, 569, 1976, pp. 124-135.

PINTON, M.R., "Socioeconomic Aspects", **Studies in Travel Demand**, vol. III, **Mathematica**, 1967, pp. 75-109.

RAE, J., **Intercity Passenger Transport in Canada**, Commission canadienne des transports, rapport 254, 1976, 65 p.

SOBIENIAK, J.W., **Forecasts of Passenger Travel in Canada's Domestic Long-Haul Air Market**, Canadian Transport Commission, report 44, 1972.

STRASZHEIM, M.R., "Airline Demand Functions in the North Atlantic and Their Pricing Implications", **Journal of Transportation Economics and Policy**, 1978, pp. 179-195.

TRAPANI, J.M. and V. OLSON, "An Analysis of the Impact of Open Entry on Price and Quality of Service in the Airline Industry", **Review of Economics and Statistics**, 1982, pp. 67-76.

VERLERGER, P.K., "Models of the Demand for Air Transportation", **Bell Journal of Economics**, 1972, pp. 437-457.

WILLS, M.J., **Estimating the Demand for Travel Using Abstract Mode Methodology**, Research Report. Ottawa 1974: Ministère d'Etat des affaires urbaines.

APPENDIX 1

Air Network

Travel time (TRAVELT):
Time of the itinerary (in minutes) on board the plane.

Transfer time (TRANSFET):
Time of the itinerary (in minutes) for the transfers, stops, airport and downtown access.

Number of departures (FREQUEN):
Number of departures for the itinerary during 1983.

Price of the trip (COST):
Price (tax included) of the itinerary.

First class (FIRST C):
Dummy variable to indicate the presence of first class paths into the itinerary.

Second class (SECND C):
Dummy variable to indicate the presence of second class paths into the itinerary.

Third class (THIRD C):
Dummy variable to indicate the presence of third class paths into the itinerary.

Two itineraries (CTE1):
Dummy variable to indicate that the market has two itineraries.

2+ itineraries (CTE2):
Dummy variable to indicate that the market has more than two itineraries.

Level of service (SIJC):
Index of the air level of service for a particular market defined by equation (4).

Network of the substitute mode

*Time car, bus (TB*TC)*:
Time (in minutes) for the shortest path by bus multiplied by the time (in minutes) for the shortest path by car (without stops).

Number de departures (FBUS):
Number of weekly departures with the shortest path by bus.

Bus fare ($BUS):
Price of the shortest path by bus.

Nights (NIGHTCAR):
Number of overnight stays with the shortest path by car.

Frequencies (others) (FCOMP):
Quasi-dummy variable for the number of departures within a market, with at most one stop or transfer, for air carriers other than Air Canada and CP Air during 1983.

Air distance (DP):
Distance (bird's flight) between two cities.

Boat (DBOAT):
Dummy variable to indicate that a trip by car or bus necessitates the use of a ferry.

Socioeconomics

Attractiveness of the destination(POPGRAV):
Influence of city j to city i = population j/ (Σk population k), where index k designates the cities closer to city i than city j.

Linguistic affinity (CULTURE):
Index of the linguistic similiraty for the cities defined as, $L_{i,j}=100-|L_i-L_j|$, where L_i (L_j) represents the English speaking population sample in city i (j);

In-migrants (MIGIAPR):
Number of in-migrants in a city coming from another Canadian province since 1976;

Out-migrants (EMIAPR):
Number of out-migrants in a city going to another canadian province since 1976;

Population (POP83):
Estimated population of the city for the year 1983;

University (UNIV):
Number of university graduates in a city divided by the number of university graduates in the province;

Income (INCOME):
Average income of the city;

Constant (CONSTANT):
Regression constant equal to 1.

APPENDIX 2

Consider the following definitions:

$$Y_1 \equiv \begin{bmatrix} T_{121} \\ \cdot \\ \cdot \\ T_{1n1} \\ \cdot \\ \cdot \\ T_{nln} \end{bmatrix} \qquad Y_2 \equiv \begin{bmatrix} T_{12} \\ \cdot \\ \cdot \\ T_{1n} \\ \cdot \\ \cdot \\ T_{nl} \end{bmatrix} \qquad Y_3 \equiv \begin{bmatrix} \ln(O_{122} / O_{121}) \\ \cdot \\ \cdot \\ \ln(O_{12t} / O_{121}) \\ \cdot \\ \cdot \\ \ln(O_{nlt} / O_{n11}) \end{bmatrix}$$

$$X_1 \equiv \begin{bmatrix} 1, & S_{12} + S_{21}, & \overbrace{X_{11},..,X_{1k}}^{O}, & \overbrace{X_{21},..,X_{2k}}^{D}, & M_{121}+M_{211}, & X_{121}, .., X_{12k} \\ . & . & . \ . \ . & . \ . \ . & . & . & . \ . \\ . & . & . \ . \ . & . \ . \ . & . & . & . \ . \\ 1, & S_{n1} + S_{1n}, & X_{n1},..,X_{nk}, & X_{11},..,X_{1k}, & M_{121}+M_{211}, & X_{121}, & .., X_{12k} \end{bmatrix}$$

$$\underset{2}{X} \equiv \begin{bmatrix} 1, & S_{12}, & \overbrace{X_{11},..,X_{1k}}^{O}, & \overbrace{X_{21},..,X_{2k}}^{D}, & M_{121}, & X_{121}, .., X_{12k} \\ . & . & . \ . \ . & . \ . \ . & . & . & . \\ . & . & . \ . \ . & . \ . \ . & . & . & . \\ 1, & S_{n1}, & X_{n1},..,X_{nk}, & X_{11},..,X_{1k}, & M_{121}, & X_{121}, .., & X_{12k} \end{bmatrix}$$

$$\underline{\Gamma} \equiv \begin{bmatrix} \ln(\Gamma_{122}) - \ln(\Gamma_{121}) \\ \cdot \quad \cdot \quad \cdot \\ \ln(\Gamma_{12t}) - \ln(\Gamma_{121}) \\ \cdot \quad \cdot \quad \cdot \\ \ln(\Gamma_{nlt}) - \ln(\Gamma_{n11}) \end{bmatrix} \qquad b_1 \equiv \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \end{bmatrix} \qquad b_2 \equiv \begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \gamma_4 \\ \gamma_5 \end{bmatrix}$$

$$\ln(\Gamma_{ijt}) \equiv \ln [(\lambda \exp(Z_{ijt}\theta) + 1)^{1/\lambda} + \mu]$$

The ANITA model can then be written as

$$\underline{Y}_1 = \underline{X}_1 b_1 + e_1 \qquad \text{(A-1)}$$

$$\underline{Y}_2 = \underline{X}_2 b_2 + e_2 \qquad \text{(A-2)}$$

$$\underline{Y}_3 = \underline{\Gamma} + e_3 . \qquad \text{(A-3)}$$

It can be shown that the results are independent of the choice of the first itinerary as reference for the definitions $\underline{\Gamma}$ and $\underline{Y}_3$.

We suppose the residual errors in (A-1) to (A-3) are i.i.d. normal. Thus the likelihood function is:

$$\mathcal{L} = -N_1 \; .5 \ln(\sigma_1^2) - .5\,\sigma_1^{-2} \left[\underline{Y}_1 - \underline{X}_1\, b_1\right]' \left[\underline{Y}_1 - \underline{X}_1\, b_1\right]$$

$$-N_2 \; .5 \ln(\sigma_2^2) - .5\,\sigma_2^{-2} \left[\underline{Y}_2 - \underline{X}_2\, b_2\right]' \left[\underline{Y}_2 - \underline{X}_2\, b_2\right]$$

$$-N_3 \; .5 \ln(\sigma_3^2) - .5\,\sigma_3^{-2} \left[\underline{Y}_3 - \underline{\Gamma}\right]' \left[\underline{Y}_3 - \underline{\Gamma}\right] . \qquad \text{(A-4)}$$

N_i denotes the number of observations for equation i. The derivative of $\mathcal{L}$ with respect to the θ's are:

$$\frac{\partial \mathcal{L}}{\partial \theta_w} = \left(\sigma_1^{-2}\, \beta_1 \left[\underline{s}_{1w} + \underline{s}_{2w}\right]' \left[(\underline{Y}_1 - \underline{X}_1\, b_1\right] + \sigma_2^{-2}\, \gamma_1 \left[\underline{s}_{1w}\right]' \left[(\underline{Y}_2 - \underline{X}_2\, b_2\right]) + \sigma_3^{-2}\, (\underline{Y}_3 - \underline{\Gamma})' \underline{W} , \qquad \text{(A-5)}$$

Vectors $\underline{s}_{1w}$ and $\underline{s}_{2w}$ are the derivatives of S_{ij} and S_{ji} with respect to the θ parameters. The variable $\underline{W}$ is one particular variable (TRAVELT, TRANSFET, COST, FREQUEN, FIRST C, SECND C, THIRD C, CTE1, CTE2) in the vector Z_{ijt} associated with the derivative parameter. More specifically,

$$\underline{W} \equiv \begin{bmatrix} (\Gamma_{121} - \mu)^{(1-\lambda)} \exp(Z_{121}\theta)\, W_{121} \\ \vdots \\ (\Gamma_{12t} - \mu)^{(1-\lambda)} \exp(Z_{12t}\theta)\, W_{12t} \\ \vdots \\ (\Gamma_{nlt} - \mu)^{(1-\lambda)} \exp(Z_{nlt}\theta)\, W_{nlt} \end{bmatrix}$$

$$\underline{s}_{1w} \equiv \begin{bmatrix} \sum_{v=1}^{n_{ij}} \exp(\theta_1 F_{12v} + \theta_2 P_{12v} + \theta_3 H_{12v} + \theta_4 D_{12v} + \theta_5)\, W_{12v}/\exp(S_{12}) \\ \vdots \\ \sum_{v=1}^{n_{ij}} \exp(\theta_1 F_{1nv} + \theta_2 P_{1nv} + \theta_3 H_{1nv} + \theta_4 D_{1nv} + \theta_5)\, W_{1nv}/\exp(S_{1n}) \\ \vdots \\ \sum_{v=1}^{n_{ij}} \exp(\theta_1 F_{nlv} + \theta_2 P_{nlv} + \theta_3 H_{nlv} + \theta_4 D_{nlv} + \theta_5)\, W_{nlv}/\exp(S_{nl}) \end{bmatrix}$$

$$\underline{s}_{2w} \equiv \begin{bmatrix} \sum_{v=1}^{n_{1j}} \exp(\theta_1 F_{21v} + \theta_2 P_{21v} + \theta_3 H_{21v} + \theta_4 D_{21v} + \theta_5)\, W_{21v}/\exp(S_{21}) \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \sum_{v=1}^{n_{1j}} \exp(\theta_1 F_{n1v} + \theta_2 P_{n1v} + \theta_3 H_{n1v} + \theta_4 D_{n1v} + \theta_5)\, W_{n1v}/\exp(S_{n1}) \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \sum_{v=1}^{n_{1j}} \exp(\theta_1 F_{1nv} + \theta_2 P_{1nv} + \theta_3 H_{1nv} + \theta_4 D_{1nv} + \theta_5)\, W_{1nv}/\exp(S_{1n}) \end{bmatrix}$$

For the ANITA model the derivatives of $\mathcal{L}$ with respect to β and σ parameters are:

$$\frac{\partial \mathcal{L}}{\partial \beta_{i\,k}} = -\sigma_i^{-2} \left(\underline{Y}_i - \underline{X}_i\, b_i \right)' \underline{X}_{i\,k}, \qquad \text{(A-6)}$$

$$\frac{\partial \mathcal{L}}{\partial \sigma_i^2} = \frac{-.5\, N_i}{\sigma_i^2} + \frac{.5}{\sigma_i^2} \left[\underline{Y}_i - \underline{X}_i\, b_i \right]' \left[\underline{Y}_i - \underline{X}_i\, b_i \right], \qquad \text{(A-7)}$$

$$\frac{\partial \mathcal{L}}{\partial \sigma_3^2} = \frac{-.5 N_3}{\sigma_3^2} + \frac{.5}{\sigma_3^2} \left[\underline{Y}_3 - \underline{\Gamma} \right]' \left[\underline{Y}_3 - \underline{\Gamma} \right], \qquad \text{(A-8)}$$

where $i=1,2$; and $X_{i\,k}$ is a vector from the k^{th} column from the $\underline{X}_i$ matrix. The likelihood function for equations (23)-(26) is:

$$\begin{aligned} \mathcal{L}^* = & -N_1 .5 \ln(\sigma_1^2) - .5 \left[\underline{Y}_1^* - \underline{X}_1^* b_1 \right]' \left[\underline{Y}_1^* - \underline{X}_1^* b_1 \right] / \sigma_1^2 \\ & - N_2 .5 \ln(\hat{\sigma}^2 - 2\sigma_1) - .5 (\hat{\sigma}^2 - 2\sigma_1^2)^{-1} \left[\underline{Y}_2^* - \underline{X}_2^* b_1 \right]' \left[\underline{Y}_2^* - \underline{X}_2^* b_1 \right] \\ & - N_3 .5 \ln(\hat{\sigma}_3^2) - .5 \left[\underline{Y}_3 - \underline{\Gamma}\, \hat{\theta} \right]' \left[\underline{Y}_3 - \underline{\Gamma}\, \hat{\theta} \right] / \hat{\sigma}_3^2 . \end{aligned} \qquad \text{(A-9)}$$

The derivatives of (A-9) are:

$$\begin{aligned} \frac{\partial \mathcal{L}^*}{\partial \sigma_1^2} = & - N_1 .5/\sigma_1^2 - .5 (\sigma_1^2)^{-2} \left[\underline{Y}_1^* - \underline{X}_1^* b_1 \right]' \left[\underline{Y}_1^* - \underline{X}_1^* b_1 \right] \\ & + N_2 (\hat{\sigma}^2 - 2\sigma_1^2)^{-1} + (\hat{\sigma}^2 - 2\sigma_1^2)^{-2} \left[\underline{Y}_2^* - \underline{X}_2^* b_1 \right]' \left[\underline{Y}_2^* - \underline{X}_2^* b_1 \right], \end{aligned} \qquad \text{(A-10)}$$

$$\frac{\partial \mathcal{L}^*}{\partial b_1} = .5(\sigma_1^2)^{-2} \left[\underline{Y}_1^* - \underline{X}_1^* b_1 \right]' \underline{X}_1^* + (\hat{\sigma}^2 - 2\sigma_1^2)^{-2} \left[\underline{Y}_2^* - \underline{X}_2^* b_1 \right]' \underline{X}_2^*. \qquad \text{(A-11)}$$

The derivatives are useful for the optimization algorithm in Fletcher and Powell (1963). In order to maximize (A-4) we computed the parameter of the variance-covariance matrix Ω with the Berndt et al. (1974) formulae:

$$\Omega = \left(\Omega^* \right)^{-1}$$

$$\Omega^* = \sum_t \begin{bmatrix} \frac{\partial \mathcal{L}_t}{\partial \theta_k} \\ \frac{\partial \mathcal{L}_t}{\partial \beta_{i\,k}} \\ \frac{\partial \mathcal{L}_t}{\partial \sigma_1^2} \\ \frac{\partial \mathcal{L}_t}{\partial \sigma_2^2} \\ \frac{\partial \mathcal{L}_t}{\partial \sigma_3^2} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial \mathcal{L}_t}{\partial \theta_k} \\ \frac{\partial \mathcal{L}_t}{\partial \beta_{i\,k}} \\ \frac{\partial \mathcal{L}_t}{\partial \sigma_1^2} \\ \frac{\partial \mathcal{L}_t}{\partial \sigma_2^2} \\ \frac{\partial \mathcal{L}_t}{\partial \sigma_3^2} \end{bmatrix}',$$